24. Two-level bosons

An ideal Bose-Einstein gas consists of non-interacting bosons of mass $m$ which have an internal degree of freedom which can be described by assuming, that the bosons are two-level atoms. Bosons in the ground state have energy $E_0 = p^2/2m$, while bosons in the excited state have energy $E_1 = p^2/2m + \Delta$, where $p$ is the momentum and $\Delta$ is the excitation energy. Assume that $\Delta \gg k_B T$.

a) Find an equation for the Bose-Einstein condensation temperature $T_c$ for this gas of two-level bosons. (Hint: this equation may not be so easy to actually solve since it involves $T_c$ and $\exp(-\beta \Delta) = \exp(-\Delta/k_B T_c)$ at the same time.)

b) Does the existence of the internal degree of freedom raise or lower the condensation temperature?

25. Fermi energy of copper

Electrons in a piece of copper metal can be assumed to behave like an ideal Fermi-Dirac gas. Copper metal in the solid state has a mass density of $9g/cm^3$. Assume that each copper atom donates one electron to the Fermi-Dirac gas. Assume the system is at $T = 0K$.

a) Compute the Fermi energy $\varepsilon_F$ of the electron gas in eV.

b) Compute the Fermi “temperature” $T_F = \varepsilon_F/k_B$.

26. Two-dimensional Fermi gas

It is experimentally possible to confine electrons to a two-dimensional system. Thus, we want to study the properties of non-interacting spin-1/2 fermions in two dimensions. The fermions have mass $m$ and are confined to a square of area $A = L^2$.

a) Calculate the density of states of this system.

b) Calculate the Fermi energy of the system as a function of the density $n = N/A$ of the fermions. Use this result to replace the fundamental constants in the density of states by the Fermi energy.

c) Show that for low temperatures the chemical potential $\mu'$ is independent of temperature.

d) Calculate the internal energy $U$ of the system as a function of the temperature $T$ and the Fermi energy $\varepsilon_F$ for low temperatures.