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Ninth Problem Set for Physics 847 (Statistical Physics II)

Winter quarter 2004

Important date: Mar 16 9:30am-11:18am final exam

Due date: Tuesday, Mar 9

24. Two-level bosons   *8 points*

An ideal Bose-Einstein gas consists of non-interacting bosons of mass m which have an internal
degree of freedom which can be described by assuming, that the bosons are two-level atoms.
Bosons in the ground state have energy $E_0 = p^2/2m$, while bosons in the excited state have
energy $E_1 = p^2/2m + \Delta$, where p is the momentum and Δ is the excitation energy. Assume
that $\Delta \gg k_B T$.

a) Find an equation for the Bose-Einstein condensation temperature T_c for this gas of two-
level bosons. (Hint: this equation may not be so easy to actually solve since it involves
T_c and $\exp(-\beta\Delta) = \exp(-\Delta/k_B T_c)$ at the same time.)

b) Does the existence of the internal degree of freedom raise or lower the condensation
temperature?

25. Fermi energy of copper   *6 points*

Electrons in a piece of copper metal can be assumed to behave like an ideal Fermi-Dirac gas.
Copper metal in the solid state has a mass density of $9g/cm^3$. Assume that each copper atom
donates one electron to the Fermi-Dirac gas. Assume the system is at $T = 0K$.

a) Compute the Fermi energy ε_F of the electron gas in eV.

b) Compute the Fermi "temperature" $T_F = \varepsilon_F/k_B$.

26. Two-dimensional Fermi gas   *10 points*

It is experimentally possible to confine electrons to a two-dimensional system. Thus, we want
to study the properties of non-interacting spin-1/2 fermions in two dimensions. The fermions
have mass m and are confined to a square of area $A = L^2$.

a) Calculate the density of states of this system.

b) Calculate the Fermi energy of the system as a function of the density $n = N/A$ of the
fermions. Use this result to replace the fundamental constants in the density of states
by the Fermi energy.

c) Show that for low temperatures the chemical potential μ' is independent of temperature.

d) Calculate the internal energy U of the system as a function of the temperature T and
the Fermi energy ε_F for low temperatures.