Department of Physics
The Ohio State University

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Eighth Problem Set for Physics 847 (Statistical Physics II)

Winter quarter 2004

Important date: Mar 16 9:30am-11:18am final exam

Due date: Tuesday, Mar 2

21. Density operator of a free particle

10 points

A free particle is described by the Hamiltonian $\hat{H} = \hat{\vec{p}}^2/2m$. We assume that the particle is in a cubic box of volume $V = L^3$.

a) Write the canonical density operator in the momentum base $|\vec{k}\rangle$, i.e., calculate $\langle\vec{k}'|\hat{\rho}|\vec{k}\rangle$. When calculating the partition function you may replace the sum by an integral.

b) Write the canonical density operator in the coordinate base $|\vec{r}\rangle$, i.e., calculate $\langle\vec{r}'|\hat{\rho}|\vec{r}\rangle$. You may again replace sums by integrals. (Hint: use $\langle\vec{k}|\vec{r}\rangle = V^{-1/2}\exp(-i\vec{k}\vec{r})$.)

22. Single energy level

10 points

We consider one energy level of a large quantum system as the subsystem which we want to describe while we summarize all the other levels of the system as the particle bath. The single particle energy of the level we picked is ε. Since there is only one level, the state of our subsystem is completely described by the kets $|n\rangle$ where n is the number of particles in our level. Since this number is not fixed, we use the grand canonical ensemble to describe this system.

a) Calculate the probability $p_n = \langle n|\hat{\rho}|n\rangle$ to find n particles in the system as a function of inverse temperature β, the level energy ε, and the fugacity $z = \exp(\beta\mu')$ for a bosonic system.

b) Calculate the average number of particles $\mathrm{Tr}\hat{\rho}\hat{N}$ in the system as a function of inverse temperature β, the level energy ε, and the fugacity $z = \exp(\beta\mu')$ for a bosonic system.

c) Calculate the probability $p_n = \langle n|\hat{\rho}|n\rangle$ to find n particles in the system as a function of inverse temperature β, the level energy ε, and the fugacity $z = \exp(\beta\mu')$ for a fermionic system.

d) Calculate the average number of particles $\mathrm{Tr}\hat{\rho}\hat{N}$ in the system as a function of inverse temperature β, the level energy ε, and the fugacity $z = \exp(\beta\mu')$ for a fermionic system.

23. Two harmonic oscillators *10 points*

Consider two indistinguishable particles inside a harmonic oscillator with frequency ω. The eigenenergies of a single particle in this oscillator are $E_n = \hbar\omega(n + 1/2)$ and we call the normalized eigenvector of the n-th eigenenergy $|n\rangle$.

a) Calculate the canonical partition function of two particles in this oscillator treating it as a classical system with discrete states.

b) Calculate the canonical partition function of two particles in this oscillator assuming that the two particles are bosons.

c) Calculate the canonical partition function of two particles in this oscillator assuming that the two particles are fermions.