Department of Physics
The Ohio State University

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Fifth Problem Set for Physics 847 (Statistical Physics II)

Winter quarter 2004

Important dates: Feb 10 10:30am-12:18pm midterm exam,
Mar 16 9:30am-11:18am final exam

Due date: Thursday, Feb 12

13. Spin correlations *8 points*

Consider a one-dimensional lattice with N lattice sites and assume that the ith lattice site has spin $s_i = \pm 1$. The Hamiltonian describing this lattice is $H = -\varepsilon \sum_{i=1}^{N} s_i s_{i+1}$. Assume periodic boundary conditions, so $s_{N+1} \equiv s_1$. Compute the correlation function $\langle s_1 s_2 \rangle$. How does it behave at very high temperature and at very low temperature?

14. Spin 1 magnet *12 points*

Consider a lattice of N spins s_i which can take values $s_i \in \{-1, 0, 1\}$. In the absence of an external magnetic field the energy of this system is given by

$$H = -\varepsilon \sum_{\{i,j\}} s_i s_j .$$

Apply the mean field approximation to this system. Denote the number of nearest neighbors of a spin by ν.

a) At which temperature T_c does the system have a phase transition?

b) How does the magnetization behave at $T > T_c$, at $T \to 0$, and at $T \approx T_c$ but $T < T_c$

c) Calculate the heat capacity in the three temperature regimes given in b).

15. Ideal gas — grand canonical ensemble *12 points*

As the simplest example for the grand canonical ensemble, we want to study the ideal gas again. The energy of an ideal gas is given by

$$H(N, \vec{x}^N) = \sum_{j=1}^{N} \frac{\vec{p}_j^2}{2m}.$$

a) Calculate the grand canonical partition function of the ideal gas.

b) Calculate the grand canonical potential of the ideal gas as a function of temperature T, volume V, and chemical potential μ'.

c) Verify the ideal gas law $PV = Nk_BT$ by taking appropriate derivatives of the grand canonical potential. (Hint: P and N are given by one derivative each.)

d) Calculate $\langle N^2 \rangle - \langle N \rangle^2$ by taking a suitable derivative of the grand canonical potential. Express the relative fluctuation $\sqrt{\langle N^2 \rangle - \langle N \rangle^2}/N$ of the number of particles as a function of temperature T, volume V, and the number of particles N. How large is this relative fluctuations for 1 mole $(6 \cdot 10^{23})$ of particles?