Department of Physics
The Ohio State University

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Third Problem Set for Physics 847 (Statistical Physics II)

Winter quarter 2004

Important dates: Feb 10 10:30am-12:18pm midterm exam,
Mar 16 9:30am-11:18am final exam

Due date: Tuesday, Jan 27

7. Ultra-relativistic gas

12 points

If the particles of a gas have velocities close to the speed of light c, their energy has to be calculated relativistically. In the limit of massless particles (e.g., photons) which travel at the speed of light, this relation between the momentum $\vec{p}_i$ of particle i and its energy E_i becomes $E_i = c|\vec{p}_i|$. Thus, the energy of a gas of N of these particles in a box of volume V is

$$H(\vec{x}^N) = \begin{cases} c\sum_{i=1}^{N} |\vec{p}_i| & \text{all } \vec{r}_i \text{ inside volume } V \\ +\infty & \text{otherwise} \end{cases} .$$

a) Calculate the partition function of such an ultra-relativistic gas. (Hint: $\int_0^\infty x^2 \exp(-x)dx = \Gamma(3) = 2$.)

b) Calculate the free energy of the ultra-relativistic gas as a function of its natural variables temperature T, volume V, and number of particles N. Use Stirling's formula to express your result in an explicitely extensive form.

c) Derive the equation of state (i.e., the relation between pressure P, volume V, temperature T, and number of particles N) of the ultra-relativistic gas.

d) Express the internal energy U of the ultra-relativistic gas in terms of the temperature T and the number of particles N.

8. Rubber elasticity

12 points

As a simple model of an elastic string like, e.g., a rubber band, we consider a linear chain of N building blocks. Each building block can be in two different states a or b. In these states the building blocks have length l_a, and l_b and energies ε_a and ε_b, respectively. The total length of the chain is $L = N_a l_a + N_b l_b$ and the total energy of the string by itself is $E_0 = N_a\varepsilon_a + N_b\varepsilon_b$ where $N_a = N - N_b$ is the number of building blocks in state a. The string is streched by an external force f which turns the total energy of a state into $E = E_0 - Lf$.

a) Calculate the partition function of this string as a function of temperature T, the number of building blocks N, and the external force f. Introduce variables $n_i \in \{a, b\}$ that describe in which state building block i is and write the partition function as a sum over these variables n_i.

b) Calculate the average internal energy U of this string as a function of temperature T, the number of building blocks N, and the external force f.

c) Calculate the expected length $\langle L \rangle$ of this string as a function of temperature T, the number of building blocks N, and the external force f. (Hint: The expected length is a similar quantity as the expected energy. Find an expression for the expected length through a derivative similar to the derivative which we use to calculate the average internal energy.) What is the expected length at zero force in the case $\varepsilon_a = \varepsilon_b$? Why?

9. Relative momentum *10 points*

We calculated in the lecture the distribution of velocities of the molecules of an ideal gas. In addition we would like to know how the *relative momentum* of two particles of the ideal gas is distributed. We concentrate on the absolut value $p_r \equiv |\vec{p}_i - \vec{p}_j|$ of this relative momentum. Calculate the probability density $P_{P_r}(p)$ of p in an ideal gas. (Hint: at some point you are left with an integral over the momenta of two different particles. It is is useful to transform this integral to a "center of mass" and a "relative" momentum in the same way as this is done in classical mechanics for coordinates, i.e., $\vec{P} = (\vec{p}_i + \vec{p}_j)/2$ and $\vec{p} = \vec{p}_i - \vec{p}_j$.)