Department of Physics
The Ohio State University

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Second Problem Set for Physics 847 (Statistical Physics II)

Winter quarter 2004

Important dates: Feb 10 10:30am-12:18pm midterm exam,
Mar 16 9:30am-11:18am final exam

Due date: Tuesday, Jan 20

4. Two-state system *8 points*

A (very small) discrete system has only two states 1 and 2 with energies $E_1 = -\varepsilon_0$ and $E_2 = \varepsilon_0$, respectively. This could, e.g., be a spin 1/2 particle in an external magnetic field. Since this system contains only one particle, the different thermodynamic ensembles do *not* provide equivalent descriptions of the physics. We want to explore this difference for this simplest possible system.

a) If the system is isolated from the environment which are the possible values for the internal energy of the system?

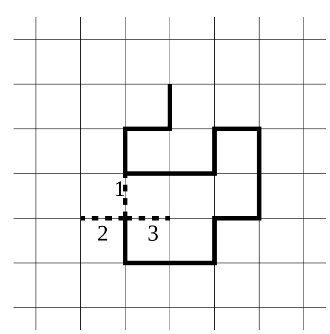
b) In the following we assume that the system is not isolated any more but instead interacting with a heat bath of temperature T. What are the probabilities p_i to find the system in each of the two states in this case.

c) Calculate the internal energy as a function of the temperature of the heat bath.

d) Which are the possible values for the internal energy of the system when coupled to the heat bath? Consider the limiting values of the expression you calculated in c).

5. A polymer on the lattice - canonical *10 points*

In order to compare the different ensembles we want to study the two dimensional polymer from problem 3 in the canconical ensemble. Again, we model the polymer as a path on a square lattice. At every lattice point the polymer can either go straight (option 1 in the figure) or choose between the two directions in a right angle with respect to its current direction (options 2 and 3 in the figure.) Each time it bends in a right angle, it pays a bending energy ϵ. If it goes straight the energy contribution of the respective joint is zero. Thus, for a given "shape" of the polymer the total bending energy of the polymer is ϵ times the number of right angle turns. We assume that the starting segment of the polymer is fixed somewhere on the lattice and that the polymer consists of $N + 1$ segments. Each possible shape of the polymer is a state of this statistical mechanics system.



a) Calculate the partition function of this polymer as a function of temperature T and the number of joints N of the polymer.

b) Calculate the average internal energy U of this polymer as a function of temperature T and the number of joints N of the polymer.

c) Compare your result in b) with the result from problem 3d).

6. Barometric formula *10 points*

We want to derive how the air pressure depends on the height above sea level. To make things simple we pretend that air consists of only one type of molecules of mass m. We consider a volume of gas over an area of size A at sea level. The z-coordinate of the molecules has to be positive (i.e., above sea level) but is otherwise unconstrained. There are N molecules in this volume. The energy of the gas is given by

$$H(\vec{x}^N) = \begin{cases} \sum_{i=1}^{N} \left[\frac{\vec{p}_i^2}{2m} + mgz_i\right] & \text{all } z_i \geq 0 \text{ and } (x_i, y_i) \text{ within area } A \\ +\infty & \text{otherwise} \end{cases},$$

where $\vec{r}_i = (x_i, y_i, z_i)$ are the coordinates of molecule i.

a) Calculate the partition function $Z(T)$ of the gas.

b) The probability density $\rho(\vec{r})$ to find *some* particle of the gas at position $\vec{r}$ is given by

$$\rho(\vec{r}) = \int d\vec{x}^N \rho(\vec{x}^N)\delta(\vec{r}_1 - \vec{r})$$

where $\vec{r}_1$ is the position of particle 1. Calculate this probability density by performing the integral.

c) Use the law of large numbers and the ideal gas law $PV = Nk_BT$ applied to the volume $V = A\Delta z$ for a small height slice of thickness Δz in order to relate the probability density $\rho(\vec{r})$ to the pressure $P(z)$ at height z.