Department of Physics
The Ohio State University

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First Problem Set for Physics 847 (Statistical Physics II)

Winter quarter 2004

Important dates: Feb 10 10:30am-12:18pm midterm exam,
Mar 16 9:30am-11:18am final exam

Due date: Tuesday, Jan 13

1. Review

10 points

Let us start by reviewing some of the key concepts from the first statistical mechanics course.

a) What are the three laws of thermodynamics?

b) What is special about the entropy in thermodynamic equilibrium compared to all other possible states of a system?

c) What is a thermodynamic phase transition and what is the difference between a first-order and a continuous phase transition?

d) What is the relation between the entropy of a macroscopic state and the number of microscopic states which correspond to the macroscopic state?

e) What is the probability distribution of the microcanonical ensemble? What physical situation does it correspond to?

2. Harmonic oscillators

10 points

Let us consider an Einstein solid and look at it from a classical point of view. The Einstein solid picture assumes that atom i of a solid sits at lattice position $\vec{r}_{0,i}$ and can oscillate harmonically around this position. It is described by the Hamiltonian

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m} + \frac{1}{2} m\omega^2 \sum_{i=1}^{N} (\vec{r}_i - \vec{r}_{0,i})^2.$$

a) What shape does the phase space volume with energy less than E, i.e., the volume $H(\vec{x}^N) \leq E$ have?

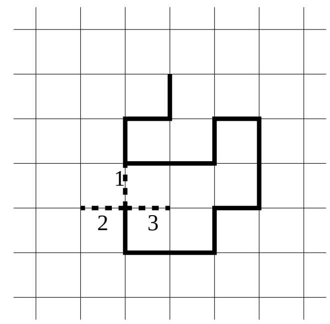
b) Calculate the entropy S of the Einstein solid at a given energy E using classical theory. When you evaluate the constant C_N take into account, that the oscillators have different origins and thus the atoms are distinguishable!

c) In what limit does your result coincide with the quantum mechanical result derived in last quarter's lecture? (You can find last quarter's lecture on the Einstein solid at `http://cannoli.mps.ohio-state.edu/phy846/phy846-14.pdf`)

d) What would the entropy be, if we had a gas of N particles within *the same* harmonic oscillator, i.e., if $\vec{r}_{0,i} = 0$ for all i. Is this entropy an extensive quantity? (Hint: You have to use a different constant C_N.)

3. A polymer on the lattice - microcanonical *10 points*

To appreciate the differences between the microcanonical and the canonical ensemble we want to solve one more problem in the microcanonical ensemble. A simple model for a polymer in two dimensions is that of a path on a square lattice. At every lattice point the polymer can either go straight (option 1 in the figure) or choose between the two directions in a right angle with respect to its current direction (options 2 and 3 in the figure.) Each time it bends in a right angle, it pays a bending energy ϵ. Thus, for a given "shape" of the polymer the total bending energy of the polymer is ϵ times the number of right angle turns. We assume that the starting segment of the polymer is fixed somewhere on the lattice and that the polymer consists of $N+1$ segments. Each possible shape of the polymer is a state of this statistical mechanics system.



a) How many polymer shapes have a total bending energy E where we assume $E = m\epsilon$ with some integer $0 \leq m \leq N$? (Hint: First count how many ways there are to position the m right angles on the polymer of length $N+1$ segments and then take into account that there are 2 possible choices for each right angle, namely left and right.)

b) What is the entropy $S(E, N)$ of this system? Approximate all factorials with the help of Stirling's formula.

c) Calculate the temperature of this system as a function of the total bending energy E and the length N of the polymer.

d) Calculate the energy E of the polymer as a function of the temperature T and of the length N of the polymer.

e) Calculate the heat capacity at constant length C_N as a function of the temperature T and the length N of the polymer.