Department of Physics
The Ohio State University

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Eighth Problem Set for Physics 846 (Statistical Physics I)

Fall quarter 2003

Important dates: Nov 27 no class, Dec 11 9:30am-11:18am final exam

Due date: Thursday, Nov 20

22. Sum of independent random variables *8 points*

The stochastic variables X and Y are independent and Gaussian distributed with first moment $\langle X \rangle = \langle Y \rangle = 0$ and standard deviations $\sigma_X = \sigma_Y = 1$. A new stochastic variable Z is constructed from these two variables by $Z = X^2 + Y^2$.

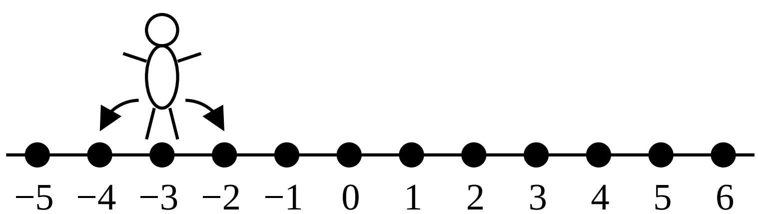
a) Find the characteristic function $f_Z(k)$ of Z.

b) Compute the moments $\langle Z \rangle$, $\langle Z^2 \rangle$, and $\langle Z^3 \rangle$ using the explicit formula for the moments of Gaussian distributions which we derived in the lecture.

c) Compute the first three cumulants from the moments.

23. Random walk *12 points*

A one dimensional random walker is a particle on a one-dimensional lattice. In each time step it chooses to jump one lattice site to the left with probability p or one lattice site to the right with probability $1-p$. Assuming that the walker starts at position 0 the position Y_N of such a random walker after N time steps can therefore be written as $Y_N = X_1 + X_2 + \ldots + X_N$ where the X_i are i.i.d. stochastic variables which take the two values -1 and 1 with probability p and $1-p$ respectively.



a) Calculate the average position $< Y_N >$ of the random walker after N steps.

b) Calculate the standard deviation σ_{Y_N} of the random walker after N steps.

c) Calculate the probability that the random walker returns to its starting point after an even number N of steps, i.e., $\Pr\{Y_N = 0\}$. (Hint: Determine how many of the possible configurations of the X_i sum up to zero and what the individual probability of these configurations is. Remember that the number of possibilities to choose a subset of size k from a set of total size N is $\binom{N}{k} = \frac{N!}{k!(N-k)!}$.)

d) Use Stirling's approximation $N! \approx \sqrt{2\pi} N^{N+1/2} e^{-N}$ to calculate the "return probability" obtained in c) in the limit of many time steps (large N.)