Department of Physics
The Ohio State University
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Sixth Problem Set for Physics 846 (Statistical Physics I)

Fall quarter 2003

Important dates: Nov 11 no class, Nov 27 no class, Dec 11 9:30am-11:18am final exam

Due date: Thursday, Nov 6

16. Boiling spaghetti in Denver *10 points*

We want to know at what temperature you would be boiling spaghetti in Denver. To this end, we will assume that the molar volume of water vapor is much larger than the molar volume of water ($v_g \gg v_l$), that water vapor can be described as an ideal gas, and that the latent heat of vaporization Δh is independent of temperature in the temperature interval $[T, T_0]$.

a) Derive the dependence of the boiling pressure $P(T)$ on the temperature T assuming that water boils at temperature T_0 at pressure P_0 by integrating the relevant differential equation.

b) The barometric formula $P = P_0 e^{-mgh/RT_{\text{air}}}$ gives the height dependence of atmospheric pressure where $m = 29g$/mol is the average mass per mole of air. At what temperature does water boil in Denver? (Use that Denver is at $h = 5,280$ft ($1609m$) and that the latent heat of vaporization of water is 10kcal/mol (42kJ/mol).)

17. Mixing *10 points*

In problem 12 we have seen that a mixture consisting of n_1 moles of species 1 and n_2 moles of species 2 is described by the Gibbs free energy

$$G(P,T,n_1,n_2) = n_1\mu_1^{(0)}(P,T) + n_2\mu_2^{(0)}(P,T) - \lambda\frac{n_1 n_2}{n_1+n_2} + n_1 RT\ln(x) + n_2 RT\ln(1-x)$$

where $x = n_1/(n_1+n_2)$ is the concentration of species 1. We have shown in problem 12a) that the chemical potential of species 2 is given by

$$\mu_2(P,T,x) = \mu_2^{(0)}(P,T) - \lambda x^2 + RT\ln(1-x).$$

a) What is the chemical potential of species 1?

b) What are the conditions on the chemical potentials for two phases, α and β, with different concentrations x_α and x_β to coexist?

c) Show that the coexistence conditions can be fulfilled with $x_\beta = 1 - x_\alpha$ at low temperatures if $\lambda < 0$ (repulsive interaction) and find the equation that determines T as a function of x_α.

d) Find the critical temperature T_c above which there is no phase separation.

18. Berthelot gas

10 points

The equation of state of a gas is given by the Berthelot equation $(P + a/Tv^2)(v - b) = RT$, where $v = V/n$ is the molar volume and a and b are constants.

a) Find values of the critical temperature T_{c}, the critical molar volume v_{c}, and the critical pressure P_{c}, in terms of a, b, and R.

b) Does the Berthelot equation satisfy the law of corresponding states?

Hint: You may want to wait with solving this problem until after Tuesday's class.