Department of Physics
The Ohio State University Prof. R. Bundschuh

Third Problem Set for Physics 846 (Statistical Physics I)

Fall quarter 2003

Important dates: Oct 21 9:30am-10:18am make-up class,
Oct 30 10:30am-12:18pm midterm exam,
Nov 11 no class, Nov 27 no class, Dec 11 9:30am-11:18am final exam

Due date: Tuesday, Oct 21, during class

6. Entropy and engines *10 points*

a) State the entropy maximum principle.

b) What is meant by the term "thermodynamic fundamental relation"? What is the significance of this fundamental representation?

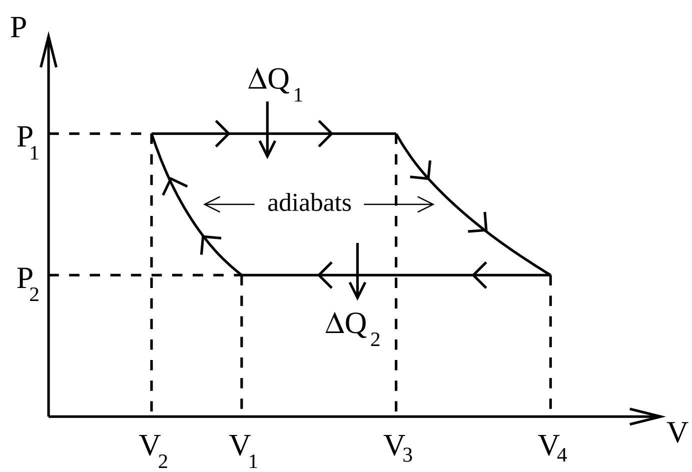
c) How are equations of state determined from the thermodynamic fundamental relation? What are the generic forms of the equations of state in the internal energy representation?

d) What are Maxwell relations? Give the Maxwell relations of a simple one-component thermodynamic system as found from the internal energy?

e) The Joule ideal gas cycle for a monoatomic ideal gas is shown below. Find the engine efficiency for this cycle in terms of P_1 and P_2.

f) Run backwards, the Joule cycle is a refrigerator. What would be a good definition of refrigerator performance? (Hint: Consider which quantity is considered the input and which quantity is considered the output for a refrigerator.)

g) Find the coefficient of refrigerator performance as a function of the engine efficiency. Will conditions that produce a large engine efficiency also produce good refrigerator performance?



7. More relations *10 points*

a) Prove that $\kappa_T(C_P - C_V) = TV\alpha_P^2$.

b) Prove that $C_P(\kappa_T - \kappa_S) = TV\alpha_P^2$.

c) Prove that $C_P/C_V = \kappa_T/\kappa_S$.

8. Internal energy of the van der Waals gas *12 points*

We have seen two examples of systems in which the internal energy depends only on temperature, namely the ideal gas which we discussed in the lecture and the magnetic system of problem 5. Now we will convince ourselves that this does not have to be the case by calculating the internal energy of the van der Waals gas.

a) Show that

$$\left(\frac{\partial U}{\partial X}\right)_T = Y - T\left(\frac{\partial Y}{\partial T}\right)_X .$$

Hint: Use the fourth general identity for partial derivatives from the lecture with $w = S$.

b) Prove the relation

$$\left(\frac{\partial C_X}{\partial X}\right)_T = \left(\frac{\partial}{\partial T}\left[Y - T\left(\frac{\partial Y}{\partial T}\right)_X\right]\right)_X .$$

Hint: dU is exact; $C_X = (\partial U/\partial T)_X$.

c) Show that the heat capacity C_V of a van der Waals gas if interpreted as a function of temperature T and volume V does not depend on the volume.

d) Calculate the internal energy $U(V,T) - U(V_0,T_0)$ under the assumption that the heat capacity C_V is constant over the temperature interval $[T_0,T]$.

e) Give a (microscopic) interpretation of the origin of the volume dependent term in your result for the internal energy. (Since you cannot really be expected to know about the microscopic picture behind the van der Waals equation at this point, you will get 2 extra points for this part.)

9. Reconstructing the equation of state *8 points*

Somebody has given you a bottle of gas and asked you to tell her what the equation of state of this substance is. As you know, the response functions, like κ_T and α_P, are the simplest quantities to measure. Thus, you went into your lab and found out that the isothermal compressibility is $\kappa_T = Tf(P)/v$ where T is the temperature, $v = V/n$ is the molar volume and $f(P)$ is some function of the pressure P which you are not so sure about after finishing your measurements. You also studied the thermal expansivity and found $\alpha_P = (R/vP) + (A/vT^2)$ with some constant A.

a) Find the function $f(P)$.

b) Find the equation of state $V = V(P,T)$ of the gas.