Department of Physics
The Ohio State University
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Second Problem Set for Physics 846 (Statistical Physics I)

Fall quarter 2003

Important dates: Oct 7 9:30am-10:18am make-up class,
Oct 30 10:30am-12:18pm midterm exam,
Nov 11 no class, Nov 27 no class, Dec 11 9:30am-11:18am final exam

Due date: Thursday, Oct 9, during class

3. Thermodynamic relations *14 points*

In this problem we want to get acquainted with the definitions of various state variables and see how relations between the state variables can be derived solely without making reference to any specific system.

a) Show that for any system
$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha_P}{\kappa_T}$$

b) Calculate the coefficient of thermal expansion α_P for an ideal gas (as a function of P and T.)

c) Calculate the compressibility κ_T of an ideal gas (as a function of P and T.)

d) Verify the general relation in a) for the ideal gas.

e) Calculate the compressibility κ_T of a van der Waals gas as a function of V and T.

f) Plot the compressibility κ_T of one mole of (i) an ideal and (ii) a van der Waals gas as a function of volume at $T = 370K$ between $V = 0.01m^3$ and $V = 0.5m^3$. Use the values $a = 0.02476\frac{Pa\cdot m^6}{mol^2}$ and $b = 0.02661\frac{m^3}{mol}$ for H_2.

4. Real numbers *8 points*

One kilogram of water is compressed isothermally at 20^oC from 1 atm to 20 atm. Assume that the average isothermal compressibility of water during this process is $\kappa_T = 0.5 \times 10^{-4}$/atm and the average thermal expansivity of water during this process is $\alpha_P = 2 \times 10^{-4}/^oC$ and that these quantities are constant during the process. If you want you may neglect small terms during your calculations.

a) How much work is required ?

b) How much heat is ejected ?

Hints: For the purposes of this problem water can be considered a "solid". You may need the Maxwell relation
$$\left(\frac{\partial S}{\partial Y}\right)_{T,\{N_j\}} = \left(\frac{\partial X}{\partial T}\right)_{Y,\{N_j\}} .$$

5. Carnot engine *12 points*

A Carnot engine uses a paramagnetic substance as its working substance. The equation of state is $\vec{M} = nD\vec{H}/T$, where $\vec{M}$ is the magnetization, $\vec{H}$ is the magnetic field, n is the number of moles, D is a constant determined by the type of the substance, and T is the temperature.

a) Show that the internal energy U if interpreted as a function of temperature T and magnetization $\vec{M}$ can only depend on the temperature and not on the magnetization.

b) Let us assume for the rest of the problem that the internal energy is simply $U = CT$ with some constant C. Sketch a typical Carnot cycle in the $\vec{M}$-$\vec{H}$-plane (you may ignore the vector character of the quantities and concentrate on their absolute values only.)

c) Compute the total heat absorbed and the total work done by the Carnot engine.

d) Compute the efficiency of the Carnot engine.

Hint: You may need the Maxwell relation

$$\left(\frac{\partial S}{\partial X}\right)_{T,\{N_j\}} = -\left(\frac{\partial Y}{\partial T}\right)_{X,\{N_j\}}.$$