Department of Physics
The Ohio State University

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Tenth Problem Set for Physics 846 (Statistical Physics I)

Fall quarter 2003

Important dates: Nov 27 no class, Dec 11 9:30am-11:18am final exam

Due date: Thursday, Dec 4

26. Spins

24 points

Consider a system composed of N non-interacting, distinguishable, spin-1/2 particles. In an applied external magnetic field H the "up" and "down" states of each spin have respectively energies $-\mu H$ and $+\mu H$ where the magnetic moment μ is a constant.

a) What is the total number of states of this system?

b) How many states are available to the system if there is an applied field H and if the system's total energy is E where $-N\mu H < E < N\mu H$ and E is an integer multiple of μH. (Hint: you may find it useful to think about the random walk problem 23 again.)

c) Write the entropy of the system as a function of E, H, and N. Make use of Stirling's formula $\ln N! \approx N \ln N - N$ to express any factorials that appear in your expressions.

d) Calculate the temperature T of the system as a function of E, H, and N.

e) Solve for the energy E as a function of the temperature T, H, and N. Checkpoint: your result should be

$$E = -N\mu H \tanh \frac{\mu H}{k_\mathrm{B} T}.$$

f) The magnetization of such a spin system is μ times the number of spins in the "up" state minus μ times the number of spins in the "down" state. Calculate the susceptibility χ_T of the system as a function of temperature T, external field H, and the number of particles N. (Hint: For this specific system of independent spins the magnetization and the energy are very closely related.)

g) Calculate the thermal expansivity α_H as a function of temperature T, external field H, and the number of particles N.

h) Express the entropy as a function of T, H, and N.

i) Calculate the heat capacity C_H of the system as a function of T, H, and N.

27. Different definitions of entropy *8 points*

Consider a system of $\mathcal{N}$ discrete states i. Each state is assigned an energy E_i. Let us for this problem assume the entropy of this system were defined as

$$S_n \equiv -\ln \sum_{i=1}^{\mathcal{N}} p_i^n$$

for some integer $n > 1$ instead of the definition $S = -k_\mathrm{B} \sum_{i=1}^{\mathcal{N}} p_i \ln p_i$ given in the lecture.

a) Show, that S_n is an extensive quantity for all n.

b) Determine which probabilities $\{p_i\}$ maximize the entropy S_n at a given energy E. (Hint: Since ln increases monotonously, maximizing the logarithm of a function is the same as maximizing the function itself.)