Problem 24:

a) $$P_{\vec{V}}(\vec{v}) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m\vec{v}^2}{2k_B T}\right)$$

$$\langle|\vec{V}|\rangle = \int \left(\frac{m}{2\pi k_B T}\right)^{3/2} |\vec{v}| \exp\left(-\frac{m\vec{v}^2}{2k_BT}\right) d^3\vec{v} \quad 2$$

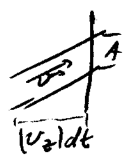
$$= 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_0^\infty v^3 \exp\left(-\frac{mv^2}{2k_BT}\right) dv$$

$$= 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \left(-\frac{d}{da}\Big|_{a=\frac{m}{2k_BT}} \int_0^\infty v e^{-av^2} dv\right)$$

$$= 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \left(-\frac{d}{da}\Big|_{a=\frac{m}{2k_BT}} \frac{1}{2}\int_0^\infty e^{-ax} dx\right)$$

$$= 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \left(-\frac{d}{da}\Big|_{a=\frac{m}{2k_BT}} \frac{1}{2a}\right)$$

$$= 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \frac{1}{2} \left(\frac{2k_BT}{m}\right)^2 = 2\sqrt{\frac{2k_BT}{m\pi}} \quad 3$$

b) 

The number of particles hitting an area A in time dt come from a volume of size $A|v_z|dt$ if they have velocity $\vec{v}$. Thus there are $\frac{N}{V} A|v_z| dt$ of them.

The total number of particles hitting the metal plate are

$$\frac{N}{V} A\, dt \int |v_z| P_{\vec{V}}(\vec{v})\, d^3\vec{v}$$

Per area and per time there are

$$2 \quad \frac{N}{V} \int_{v_z>0} |v_z| \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m\vec{v}^2}{2k_BT}\right) d^3\vec{v}$$

$$= \frac{N}{V} \int_0^\infty |v_z| \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{mv_z^2}{2k_BT}\right) dv_z$$

$$= \frac{N}{V} \int_0^\infty v_z \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left[-\frac{m v_z^2}{2 k_B T}\right] dv_z$$

$$= \frac{N}{V}\left(\frac{k_B T}{2m\pi}\right)^{1/2} \int_0^\infty e^{-x}\,dx = \frac{N}{V}\left(\frac{k_B T}{2m\pi}\right)^{1/2} \quad 2$$

c) Only those particles stick to the metal plate that have a normal velocity larger than v_T. The number of these particles per area and per unit time on one side is

$$\frac{N}{V} \int_{v_T}^\infty v_z \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left[-\frac{m v_z^2}{2 k_B T}\right] dv_z \quad 2$$

$$= \frac{N}{V}\left(\frac{k_B T}{2m\pi}\right)^{1/2} \int_{\frac{m v_T^2}{2 k_B T}}^\infty e^{-x}\,dx = \frac{N}{V}\left(\frac{k_B T}{2m\pi}\right)^{1/2} e^{-\frac{m v_T^2}{2 k_B T}} \quad 1$$

Problem 25:

a) At a fixed number of n_{-1}, n_0, and n_{+1}, there are $\left(\frac{N!}{n_1!\, n_0!\, n_{-1}!}\right)$ states 1 (with $N = n_{-1} + n_0 + n_{+1}$)

The corresponding entropy is

$$S = k_B \ln\left(\frac{N!}{n_1!\, n_0!\, n_{-1}!}\right) \approx k_B\left[N\ln N - N - n_1 \ln n_1 + n_1 - n_0 \ln n_0 + n_0 - n_{-1}\ln n_{-1} + n_{-1}\right]$$

$$= k_B\left[N\ln N - n_1\ln n_1 - n_0 \ln n_0 - n_{-1}\ln n_{-1}\right]$$ 2

b) Introduce $x_1 = \frac{n_1}{N}$ and $x_{-1} = \frac{n_{-1}}{N}$. Then $n_0 = N(1 - x_1 - x_{-1})$

$$S \approx k_B\left[\cancel{N\ln N} - \cancel{Nx_1 \ln N} - Nx_1 \ln x_1 - \cancel{N(1-x_1-x_{-1})\ln N} - N(1-x_1-x_{-1})\ln(1-x_1-x_{-1}) - \cancel{Nx_{-1}\ln N} - Nx_{-1}\ln x_{-1}\right]$$

$$= -k_B N\left[x_1 \ln x_1 + (1-x_1-x_{-1})\ln(1-x_1-x_{-1}) + x_{-1}\ln x_{-1}\right]$$

The configuration that maximizes S fulfills

$\left(\frac{\partial S}{\partial x_1}\right)_{x_{-1},N} = 0$ and $\left(\frac{\partial S}{\partial x_{-1}}\right)_{x_1,N} = 0$ 1

$$0 = \left(\frac{\partial S}{\partial x_1}\right)_{x_{-1},N} = -k_B N\left[\ln x_1 + 1 - \ln(1-x_1-x_{-1}) - 1\right]$$

$$0 = \left(\frac{\partial S}{\partial x_{-1}}\right)_{x_1,N} = -k_B N\left[-\ln(1-x_1-x_{-1}) - 1 + \ln x_{-1} + 1\right]$$ 2

$$\Rightarrow \quad x_1 = x_{-1} = 1 - x_1 - x_{-1} \quad \Rightarrow \quad x_1 = 1 - 2x_1 \Rightarrow x_1 = x_{-1} = \frac{1}{3}$$ 1

c) $S_{max} = -k_B N\left(\frac{1}{3}\ln\frac{1}{3} + \frac{1}{3}\ln\frac{1}{3} + \frac{1}{3}\ln\frac{1}{3}\right) = k_B N \ln 3$ 1