Problem 19:

Continuous transition

$\Rightarrow$ along the λ-line $P(T)$ there is no jump in entropy, i.e.,

$$S^{I}(P(T),T) = S^{II}(P(T),T) \quad (2)$$

$$\frac{d}{dT} \text{ " } : \quad \underbrace{\left(\frac{\partial S^{I}}{\partial P}\right)_T}_{-\left(\frac{\partial v^{I}}{\partial T}\right)_P} \left(\frac{dP}{dT}\right)_{coex} + \left(\frac{\partial S^{I}}{\partial T}\right)_P = \left(\frac{\partial S^{II}}{\partial P}\right)_T \left(\frac{dP}{dT}\right)_{coex} + \left(\frac{\partial S^{II}}{\partial T}\right)_P \quad (2)$$

$$-v\, \overset{(1)}{\alpha_P^{I}} \left(\frac{dP}{dT}\right)_{coex} + \frac{1}{T} \overset{(1)}{C_P^{I}} = -v\, \alpha_P^{II} \left(\frac{dP}{dT}\right)_{coex} + \frac{1}{T} C_P^{II}$$

$$-v\, \Delta\alpha_P \left(\frac{dP}{dT}\right)_{coex} + \frac{1}{T} \Delta C_P = 0$$

$$\left(\frac{dP}{dT}\right)_{coex} = \frac{\Delta C_P}{T v \Delta\alpha_P} \quad (2)$$

Problem 20:

(2) a) No heads $= \{TTT\}$ $\quad P(\{TTT\}) = \frac{1}{8}$

(2) b) At least one head $= \{TTH, THT, THH, HTT, HTH, HHT, HHH\}$

$P(\{TTH, THT, THH, HTT, HTH, HHT, HHH\}) = \frac{7}{8}$

(2) c) At least two heads $= \{THH, HTH, HHT, HHH\}$

$P(\{THH, HTH, HHT, HHH\}) = \frac{1}{2}$

d) Heads on the first coin: $H_1 \equiv \{HHH, HTH, HTT, HHT\}$

tails on the ~~last~~ coin: $T_3 = \{HHT, HTT, THT, TTT\}$

$P(H_1) = \frac{1}{2}$ $\quad P(T_3) = \frac{1}{2}$ $\quad H_1 \cap T_3 = \{HHT, HTT\}$ (1)

$P(H_1 \cap T_3) = \frac{1}{4} = P(H_1) \cdot P(T_3)$ (1)

e) exactly two coins head: $A = \{HTH, HHT, THH\}$

three coins head: $B \equiv \{HHH\}$

(1) $\quad A \cap B = \phi \Rightarrow$ mutually exclusive

$P(A) = \frac{3}{8}$ $\quad P(B) = \frac{1}{8}$

$P(A \cap B) = P(\phi) = 0 \neq \frac{3}{64} = P(A)\,P(B)$ (1)

Problem 21

(2) a) Normalization $\Rightarrow \int_{-\infty}^{\infty} p_X(x) = 1$

$$\Rightarrow 1 = \int_0^{\infty} C e^{-\lambda x} dx = C \left[\frac{1}{\lambda} e^{-\lambda x} \right]_0^{\infty} = \frac{C}{\lambda} \Rightarrow C = \lambda$$

(3) b) $f_X(k) = \int_{-\infty}^{\infty} e^{ikx} p_X(x) dx = \lambda \int_{0}^{\infty} e^{ikx} e^{-\lambda x} dx$

$$= \frac{\lambda}{ik - \lambda} \left[e^{ikx - \lambda x} \right]_0^{\infty} = \frac{\lambda}{\lambda - ik}$$

(3) c) $f_X(k) = \frac{\lambda}{\lambda - ik} = \frac{1}{1 - ik/\lambda} = \sum_{n=0}^{\infty} \frac{(ik)^n}{\lambda^n}$

$$\sum_{n=0}^{\infty} \frac{(ik)^n}{n!} \langle X^n \rangle \Rightarrow \frac{\langle X^n \rangle}{n!} = \frac{1}{\lambda^n} \Rightarrow \langle X^n \rangle = \frac{n!}{\lambda^n}$$

(4) d) $\ln f_X(k) = \ln \frac{1}{1 - ik/\lambda} = -\ln(1 - ik/\lambda) = + \sum_{n=1}^{\infty} \frac{(ik)^n}{n \lambda^n}$

$$\sum_{n=1}^{\infty} \frac{(ik)^n}{n!} C_n(X) \Rightarrow \frac{C_n(X)}{n!} = \frac{1}{n \lambda^n} \Rightarrow C_n(X) = \frac{(n-1)!}{\lambda^n}$$