oblem 10:

$$\chi_{T,n} = \left(\frac{\partial M}{\partial H}\right)_{T,n} = \frac{nD}{T} \qquad 2$$

$$\alpha_{H,n} = \left(\frac{\partial M}{\partial T}\right)_{H,n} = -\frac{nDH}{T^2} = -\frac{M}{T} \qquad 2$$

$$C_{H,n} - C_{M,n} = \frac{T\alpha_{H,n}^2}{\chi_{T,n}} = \frac{T n^2 D^2 H^2 T}{T^4 \cancel{nD}} = \frac{nDH^2}{T^2}$$

$$\Rightarrow C_{H,n} = nc + n\frac{DH^2}{T^2} = nc + \frac{M^2}{nD} \qquad 2$$

$$\frac{C_{H,n}}{C_{M,n}} = 1 + \frac{DH^2}{cT^2} = \frac{\chi_{T,n}}{\chi_{S,n}} = \frac{nD}{T\chi_{S,n}}$$

$$\Rightarrow \chi_{S,n} = \frac{nD}{T + \frac{DH^2}{cT}} = \frac{nD}{T + \frac{TM^2}{Dcn^2}} \qquad 2$$

Problem 11:

a) $$\left(\frac{\partial L}{\partial T}\right)_J \left(\frac{\partial T}{\partial J}\right)_L \left(\frac{\partial J}{\partial L}\right)_T = -1$$

$$\Rightarrow \left(\frac{\partial L}{\partial T}\right)_J = -\frac{\left(\frac{\partial J}{\partial T}\right)_L}{\left(\frac{\partial J}{\partial L}\right)_T} = -\frac{L}{T}\,\frac{1-\left(\frac{L_0}{L}\right)^3}{1+2\left(\frac{L_0}{L}\right)^3} \qquad 2$$

If the rubber band is under positive tension, i.e. $L > L_0$, the length at constant tension decreases with temperature. If it is under negative tension, the length increases with temperature. 1

b) $$dJ = \left(\frac{\partial J}{\partial L}\right)_T dL + \left(\frac{\partial J}{\partial T}\right)_L dT$$

$$\left(\frac{\partial}{\partial T}\left(\frac{\partial J}{\partial L}\right)_T\right)_L = \frac{a}{L_0}\left(1+2\left(\frac{L_0}{L}\right)^3\right)$$

$$\left(\frac{\partial}{\partial L}\left(\frac{\partial J}{\partial T}\right)_L\right)_T = \frac{a}{L_0}\left[1-\left(\frac{L_0}{L}\right)^3\right] + \frac{aL}{L_0}\,\frac{3L_0^3}{L^4}\Big) = \frac{a}{L_0}\left(1+2\left(\frac{L_0}{L}\right)^3\right) \qquad 2$$

$$J(L,T) = \int_{T_0}^{T} \frac{aL_0}{L_0}\left(1-\left(\frac{L_0}{L_0}\right)^3\right)dT + \int_{L_0}^{L} \frac{aT}{L_0}\left(1+2\left(\frac{L_0}{L}\right)^3\right)dL$$

$$= 0 + \frac{aT}{L_0}(L-L_0) - \frac{aT}{L_0}L_0^3\left(\frac{1}{L^2}-\frac{1}{L_0^2}\right)$$

$$= aT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right) \qquad 2$$

c) $$\left(\frac{\partial U}{\partial L}\right)_T = J - T\left(\frac{\partial J}{\partial T}\right)_L = J - T\,\frac{aL}{L_0}\left(1-\left(\frac{L_0}{L}\right)^3\right)$$

$$= aT\left[\frac{L}{L_0} - \frac{L_0^2}{L^2}\right] - aT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right) = 0 \quad (2)$$

d)

adiabatic $\Rightarrow$ $\underset{\substack{\| \; c) \\ C_L dT}}{dU} = -dW = J\,dL$

$$C_L dT = aT\left[\frac{L}{L_0} - \frac{L_0^2}{L^2}\right] dL$$

$$C_L\left[\ln T - \ln T_0\right] = a\left[\frac{1}{2}\frac{L^2}{L_0} + \frac{L_0^2}{L} - \frac{3}{2}L_0\right]$$

$$C_L \ln T/T_0 = a\left[\frac{1}{2}\frac{L^2}{L_0} + \frac{L_0^2}{L} - \frac{3}{2}L_0\right] = 10^{-2}\frac{N}{K}\left(1 + \frac{1}{4} - \frac{3}{4}\right]m = \cancel{0.5m}$$

$$5 \cdot 10^{-3}\,\frac{Nm}{K}$$

$$\ln T/T_0 = \frac{5 \cdot 10^{-3}\ J/K}{1\ J/K} = 5 \cdot 10^{-3}$$

$$T = T_0\, e^{5\cdot 10^{-3}} \approx 291.5\ K \quad (2)$$

$$\Delta W = \Delta U = C_L \Delta T = 1\frac{J}{K} \cdot 1.5K = 1.5J \quad (1)$$

Problem 12:

make sure that these terms are not forgotten!

a) $\mu_w(P,T,x_s) = \left(\dfrac{\partial G}{\partial n_w}\right)_{P,T,n_s}$

$$= \mu_w^{(0)}(P,T) - \frac{\lambda n_s}{n_s+n_w} + \frac{\lambda n_s n_w}{(n_s+n_w)^2} + RT\ln(1-x_s)$$

$$+ n_w RT \frac{1}{1-x_s}\,\frac{+n_s}{(n_s+n_w)^2} + n_s RT \frac{1}{x_s}\,\frac{-n_s}{(n_s+n_w)^2}$$

$$= \mu_w^{(0)}(P,T) - \frac{\lambda[n_s^2 + n_s n_w - n_s n_w]}{(n_s+n_w)^2} + RT\ln(1-x_s)$$

$$\cancel{+ RT\left[\frac{+n_s \cancel{n_w}\,\cancel{(n_s+n_w)}}{\cancel{n_w}\,(n_s+n_w)^{\cancel{2}}} - \frac{n_s^{\cancel{2}}\,\cancel{(n_s+n_w)}}{\cancel{n_s}\,(n_s+n_w)^{\cancel{2}}}\right]}$$

$$= \mu_w^{(0)}(P,T) - \lambda x_s^2 + RT\ln(1-x_s) \qquad 3$$

b) $$\mu_w(P,T,x_s) \simeq \mu_w^{(0)}(P,T) - RT x_s + O(x_s^2) \qquad 2$$

c) $$\mu_w^{(0)}(P_0,T) = \mu_w(P,T,x_s) \qquad 1$$

d) $$V = \left(\frac{\partial G}{\partial P}\right)_T = n_w\left(\frac{\partial \mu_w^{(0)}(P,T)}{\partial P}\right)_T + n_s\left(\frac{\partial \mu_s^{(0)}(P,T)}{\partial P}\right)_T$$

$$\simeq n_w\left(\frac{\partial \mu_w^{(0)}(P,T)}{\partial P}\right)_T$$

assume V constant from P_0 to P

$$\Rightarrow \quad \mu_w^{(0)}(P,T) - \mu_w^{(0)}(P_0,T) \simeq \frac{V}{n_w}(P-P_0) = \frac{V}{n_w}\pi \qquad 2$$

e) $$\mu_w^{(0)}(P_0,T) \simeq \mu_w^{(0)}(P,T) - RT x_s$$

$$RT x_s = \frac{V}{n_w}\pi \quad \Rightarrow \quad \boxed{V\pi \simeq n_s RT} \qquad 2$$