II.1 Examples

Phase transition: Through a change of external parameters, a substance changes from one phase to another.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>Gas</td>
</tr>
<tr>
<td>Liquid</td>
<td>Solid</td>
</tr>
<tr>
<td>Conductor</td>
<td>Superconductor</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ferromagnet</th>
<th>Paramagnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desaturated</td>
<td>Globular</td>
</tr>
<tr>
<td>Gas</td>
<td>Bose-Einstein-condensate</td>
</tr>
<tr>
<td>Absorbed</td>
<td>Desorbed</td>
</tr>
<tr>
<td>Isotropic</td>
<td>Nematic</td>
</tr>
</tbody>
</table>

II.2 Gibbs phase rule

Single component system:

Three degrees of freedom, say \( N, Y, T \)

At phase transition:

Two phase, coexistence:

\[ Y^1 = Y^2, \quad T^1 = T^2, \quad \mu^1 = \mu^2 \]
**DeBahn relation**

\[ \mu^1 = \mu^2 (Y, T), \quad \mu^3 = \mu^4 (Y, T) \]

two completely different functions

\( \Rightarrow \) condition for phase coexistence of two phases

\[ \mu^1 (Y, T) = \mu^4 (Y, T) \]

reduce number of degrees of freedom by one

\( \Rightarrow \) can solve for \( Y = Y(T) \) "coexistence curve; one intensive degree of freedom"

**Three phase coexistence**

\[ Y^1 = Y^2 = Y^{co} \Rightarrow Y, \quad T^1 = T^2 > T^{co} \Rightarrow T \]

\[ \mu^1 (Y, T) = \mu^2 (Y, T) = \mu^{co} (Y, T) \]

reduce number of degrees of freedom by two

\( \Rightarrow \) no intensive degree of freedom

\( \Rightarrow \) \( (Y^{co}, T) \) isolated points

**Four phase coexistence**

\[ \mu^3 (Y, T) = \mu^2 (Y, T) = \mu^{co} (Y, T) = \mu^{id} (Y, T) \]

three equations for two unknowns

\( \Rightarrow \) four phases cannot coexist in a single component system
general rule for multi-concurrent system

\( l \) concurrents

\( \rightarrow l + 1 \) intensive degrees of freedom in each phase

\[ Y, T, x_1, x_2, \ldots, x_{l-1}, \quad x_i = \frac{R_i}{\sum_{j=1}^{l} R_j} \]

coexistence of \( r \) phases:

\( Y, T \) the same in all phases, but \( x_i \) can be different

\( \Rightarrow 2 + (l-1) \) intensive degrees of freedom

coexistence condition

\[ \mu_1^I (Y, T, x_1^I, \ldots, x_{l-1}^I) = \mu_2^I (Y, T, x_1^I, \ldots, x_{l-1}^I) = \cdots = \mu_r^I (Y, T, x_1^I, \ldots, x_{l-1}^I) \]

\[ \mu_1^II (Y, T, x_1^{II}, \ldots, x_{l-1}^{II}) = \mu_2^II (Y, T, x_1^{II}, \ldots, x_{l-1}^{II}) = \cdots = \mu_r^II (Y, T, x_1^{II}, \ldots, x_{l-1}^{II}) \]

\[ \mu_1^I (Y, T, x_1^I, \ldots, x_{l-1}^I) = \mu_2^I (Y, T, x_1^I, \ldots, x_{l-1}^I) = \cdots = \mu_r^I (Y, T, x_1^I, \ldots, x_{l-1}^I) \]

\( l \) \((r-1)\) equations

\( \Rightarrow \) A \( r \)-phase coexistence region of an \( l \)-component system has

\[ 2 + (l-1) - l(r-1) = 2 + l - r \]

intensive degrees of freedom

"Gibbs phase rule"