
Second Problem Set for Physics 664 (Theoretical Mechanics)

Spring quarter 2004

Important dates: Apr 27 9:30am-10:18am & May 11 9:30am-10:18am midterm exams,
May 31 no class, Jun 9 9:30am-11:18am final exam

Due date: Thursday, Apr 15

4. Overdamped oscillations

10 points

If a harmonic oscillator with friction is in the overdamped regime it performs the motion

$$x(t) = e^{-\beta t} [A_1 e^{\omega_2 t} + A_2 e^{-\omega_2 t}]$$

where $\omega_2 \equiv \sqrt{\beta^2 - \omega_0^2}$ and A_1 and A_2 are real constants determined by the initial conditions.

- Calculate the velocity $\dot{x}(t)$.
- Determine A_1 and A_2 as functions of the position x_0 and velocity v_0 of the motion at $t = 0$.
- Discuss the geometric shape of the phase path of the motion in the case $A_1 = 0$.
- Discuss the asymptotic behavior of the phase paths of the motion in the case $A_1 \neq 0$ for $t \rightarrow \infty$.

5. Phase portrait

12 points

A particle of mass m moves along the x axis under the influence of the potential

$$V(x) = -\frac{1}{x^4 + ax^2 + 2}$$

where a is a dimensionless constant.

- Sketch the potential for $a = 2$ and $a = -2$.
- Sketch the phase portraits of the system for the cases $a = 2$ and $a = -2$.
- Indicate stable and instable equilibrium points and the separatrices in these phase portraits if they exist. Annotate the separatrices with the energies they correspond to.

6. Fourier expansion

12 points

We want to consider the periodic driving force

$$F(t) = \begin{cases} -1 & t \in \left] \frac{2\pi}{\omega}n - \frac{\pi}{\omega}, \frac{2\pi}{\omega}n \right[\text{ for some integer } n \\ +1 & t \in \left] \frac{2\pi}{\omega}n, \frac{2\pi}{\omega}n + \frac{\pi}{\omega} \right[\text{ for some integer } n \end{cases}.$$

To this end we have to find the Fourier expansion of this function.

- a) Sketch $F(t)$.
- b) Obtain the Fourier expansion

$$F(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

of $F(t)$.

- c) Plot the sums of the first two terms, first three terms, and first four terms in the interval $[-2\pi/\omega, 2\pi/\omega]$ for $\omega = 1\text{rad/s}$.